Numerical analysis: Homework 5

Instructor: Anil Damle

Due: April 26, 2024

Policies

You may discuss the homework problems freely with other students, but please refrain from looking at their code or writeups (or sharing your own). Ultimately, you must implement your own code and write up your own solution to be turned in. Your solution, including plots and requested output from your code should be typeset and submitted via the Gradescope as a pdf file. This file must be self contained for grading. Additionally, please submit any code written for the assignment as zip file to the separate Gradescope assignment for code.

Question 1:

Consider the penalized formulation for solving a quadratic problem with equality constraints

$$\min_x \frac{1}{2}x^T Hx + x^T c + \frac{1}{2\mu}\|Ax - b\|_2^2, \tag{1}$$

where $H \in \mathbb{R}^{n\times n}$ is symmetric, $c \in \mathbb{R}^n$, $\mu > 0$, and $A \in \mathbb{R}^{m\times n}$ with $m < n$ and full row-rank. Let $Z \in \mathbb{R}^{n\times(n-m)}$ be a matrix with orthonormal columns whose range is the null space of A. Show that if $Z^T HZ$ is positive definite, then for all sufficiently small μ (1) has a unique solution and that solution is a strict local minimizer.