CS 1115: Final Exam

December 7, 2012

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(Name)

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(Cornell ID)

Problem 1	15 points	
Problem 2	15 points	
Problem 3	15 points	
Problem 4	10 points	
Problem 5	15 points	
Problem 6	15 points	
Problem 7	15 points	

The exam has nine (9) different pages counting this one. Confirm this.

1. In this problem you are not allowed to use any MATLAB trig function or inverse trig function. Recall the trigonometric identities

$$
\begin{aligned}
\sin(a+b) &= \sin(a)\cos(b) + \cos(a)\sin(b) \\
\cos(a+b) &= \cos(a)\cos(b) - \sin(a)\sin(b) \\
\sin(a/2) &= \sqrt{(1-\cos(a))/2} \qquad 0 \le a \le \pi/2 \\
\cos(a/2) &= \sqrt{(1+\cos(a))/2} \qquad 0 \le a \le \pi/2 \\
\sin(-a) &= -\sin(a) \\
\cos(-a) &= \cos(a)
\end{aligned}
$$

Assume that the variables `s` and `c` contain the sine and cosine of some angle θ that satisfies $0 < \theta < \pi/2$.

(a) Write a script that assigns $\sin(3\theta/4)$ to `s2` and $\cos(3\theta/4)$ to `c2`. Use the fact that $\sin(0) = 0$ and $\cos(0) = 1$.

(b) Write a script that assigns $\sin(\alpha\theta)$ to `st` and $\cos(\alpha\theta)$ to `ct` where

$$
\alpha = \frac{2^t - 1}{2^t}
$$

and it is assumed that `t` is an initialized positive integer. Your solution must not make use of arrays.

2. Assume that the following fragment plays a message:

```
[v,f] = wavread('MyMessage);
sound(v,f)
```

Assume that `n` is a positive integer. Write a MATLAB fragment that computes a vector `y` such that

```
sound(y,f)
```

plays the message n times with $n-1$ intervals of silence in between the repetitions. The kth interval of silence should last approximately $T/2^k$ seconds where T is the duration of `sound(v,f)`.

```
[Y,FS] = WAVREAD(FILE) returns the sample data in columnn vector Y and the sample rate in FS

SOUND(Y,FS) sends the signal in column vector Y (with sample frequency FS) out
        to the speaker on platforms that support sound. Values in
        Y are assumed to be in the range -1.0 <= y <= 1.0.
```

3. Complete the following function so that it performs as specified:

```
 function A = CellMax(C)
% C is a length-p cell array of m-by-n matrices
% A is an m-by-n matrix and A(i,j) is the maximum of the (i,j)
% entries in C{1},...,C{p}.
```

Thus, if

```
 C = { [ 1 2 ; 3 9 ; 5 6] , [ -1 3 ; 4 -1 ; 6 7]}
```

then

```
 A = [ 1 3; 4 9 ; 6 7]
```

```
    MAX    Largest component.
           For vectors, MAX(X) is the largest element in X.
```

4. Assume the availability of the the following class:

```
classdef Point < handle
% A point has an x and y coordinate
  properties
      x
      y
  end

  methods
     function P = Point(x,y)
        P.x = x;
        P.y = y;
     end

     function d = Dist(ThisPoint,Q)
        % The distance from ThisPoint to the point referenced by Q.
        ...
     end
  end
end
```

(a) Would this be a way to display the coordinates of a point that is midway between (0,0) and (a, b) assuming that `a` and `b` are initialized? Explain.

```
P = Point(a,b);
Q = P/2;
Qx = Q.x
Qy = Q.y
```

(b) Write an instance method that can be used to compute a point object that encodes the point that is midway between two given points.

(c) Assume the availability of the `Point` class above and

```
classdef CirclePolygon  < handle

% A CirclePolygon object represents a polygon whose vertices are on a circle.

properties
    C = Point.empty();  % The center of the circle
    r                   % The radius of the circle
    V = Point.empty();  % A vector of points that encodes the vertices
end

methods

    function P = CirclePolygon(C,r,theta)
    % C is a point, r is a positive real number, and theta is a length-n vector
    %      with the property that 0<=theta(1)< theta(2)<...< theta(n) < 2*pi.
    % P is a circle polygon with center C , radius r
    % and vertices (C.x + r*cos(theta(k)),C.y + r*sin(theta(k))) for k=1:n.
        ...
    end
end
```

Write an instance method for this class that can be used to compute the perimeter of the polygon obtained by connecting the midpoints of a given circle polygon. You must take full advantage of the the `Point` class to receive full credit.

5. (a) Assume that the availability of

```
   function A = AreaCodes()
% A is a 500-by-3 character array that encodes all valid 3-digit area codes.
```

Develop an efficient implementation of the following function:

```
 function alfa = IsAC(s)
% s is a length-3 string
% alfa is 1 if s encodes a valid area code and 0 if it does not.
```

(b) A string **s** encodes a valid telephone number if it equals '**911**', or if it has the form '**1xxxyyyzzzz**' where **xxx** is a valid area code and **yyy** is not a valid area code, or if it has the form '**yyyzzzz**' where '**yyy**' is not a valid area code. Otherwise, **s** does not encode a valid telephone number. Complete the following function so that it performs as specified.

```
  function alfa = IsNumber(s)
% s is a string of digit characters.
% alfa is 1 if s encodes a valid telephone number and is 0 otherwise
```

You may assume the availability of the function **ISAC** from part (a).

```
STRCMP Compare strings.
    TF = STRCMP(S1,S2) compares the strings S1 and S2 and returns logical 1
    (true) if they are identical, and returns logical 0 (false) otherwise.
```

6.(a) Assume the availability of the following function:

```
function DrawSquares(a,b,s,n)
if n==0
    fill([a a+s a+s a],[b b b+s b+s],'m')
else
    s = s/2;
    DrawSquares(a+s,b,s,n-1)
    DrawSquares(a,b+s,s,n-1)
    DrawSquares(a+s,b+s,s,n-1)
end
```

Sketch the output that would result if the following script is run:

```
plot([0 1 1 0 0],[0 0 1 1 0],'k')
hold on
DrawSquares(0,0,1,3)
```

Your sketch should shade those tiles that are magenta and leave white those tiles that are not magenta.

(b) Assume that n is a positive integer. As a function of n, what fraction of the unit square is colored magenta by the command `DrawSquares(0,0,1,n)`? Briefly explain your reasoning.

7. We say that an integer is a "good" integer if it has the form $2^i 3^j$ for some $i \geq 0$ and $j \geq 0$. Thus, $2^0 3^0 = 1$ and $2^5 3^4 = 32 \cdot 81 = 2592$ are examples. Assume that M is an initialized variable with positive value M. Write an efficient MATLAB script that assigns to G the number of good integers that are strictly less than M. Thus, if $M = 50$, then the value of G should be 15 since the numbers

$$
\begin{array}{llll}
2^0 3^0 & 2^0 3^1 & 2^0 3^2 & 2^0 3^3 \\
2^1 3^0 & 2^1 3^1 & 2^1 3^2 \\
2^2 3^0 & 2^2 3^1 & 2^2 3^2 \\
2^3 3^0 & 2^3 3^1 \\
2^4 3^0 & 2^4 3^1 \\
2^5 3^0
\end{array}
$$

are all less than 50. Your are not allowed to use any built-in functions in your solution, e.g., $\texttt{log}$. For your information, if $2^{i_1} 3^{j_1} = 2^{i_2} 3^{j_2}$, then $i_1 = j_1$ and $i_2 = j_2$.