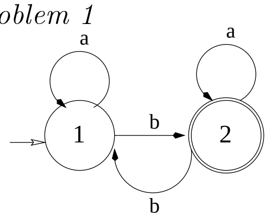
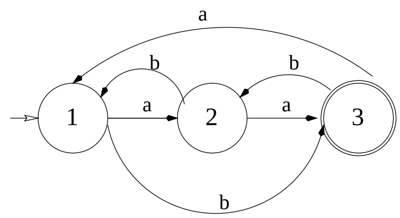
CS 381 Fall 2000
Solutions to Homework 2

Handout #4

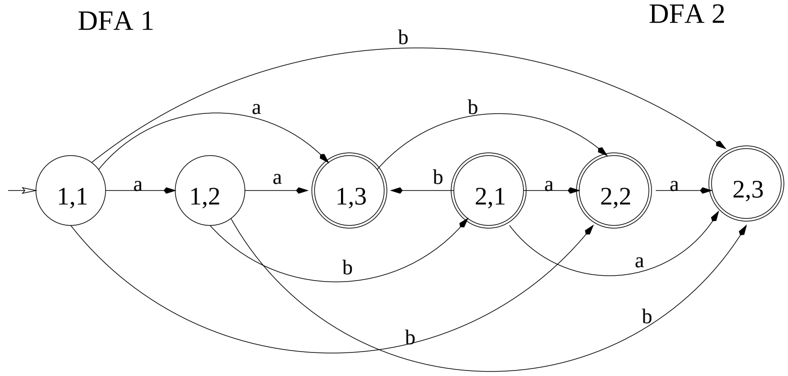
Problem 1



DFA 1



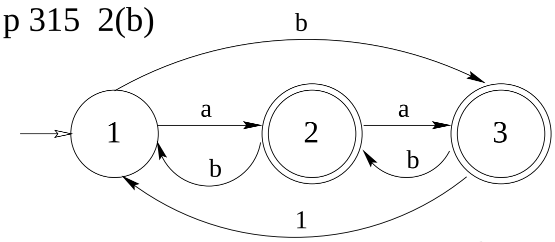
DFA 2



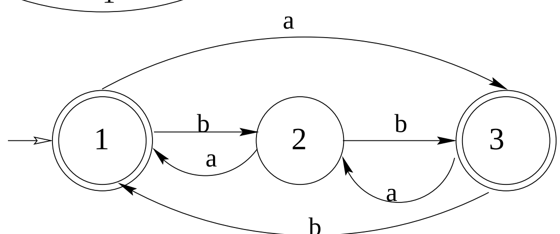
UNION OF DFAs 1 & 2

In case of intersection (2,3) is the only final state .

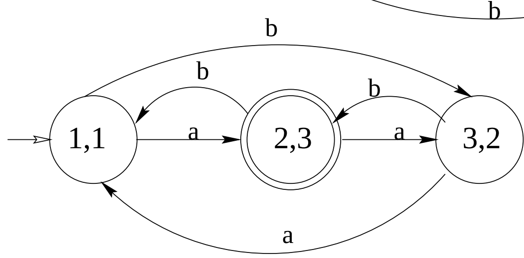
p 315 2(b)



DFA 1



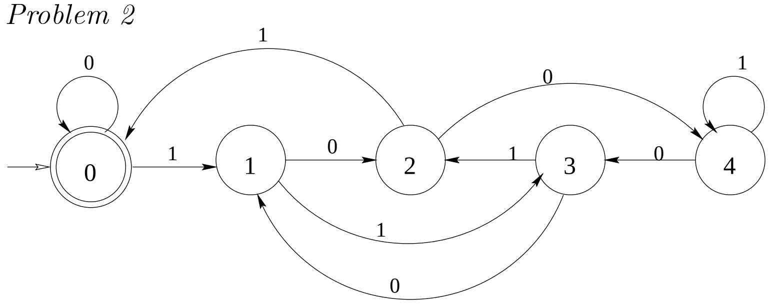
DFA 2



Intersection of DFAs 1 & 2

In union, all the states are final states

Problem 2



DFA accepting $A_{5,2}$

Proof that the DFA accepts $A_{5,2}$

Claim : If $w = a_1a_2 \ldots a_n$ and $decimal(w) = 5k + r$, $0 \leq r < 5$,where $decimal(w)$ is the decimal value represented by the binary string , then the above DFA after reading w is in state r.

Proof of the claim by induction on length of w: If $|w|$=1 , then it obvious that the claim is true. Assume that it is true for $|w| < n$. Consider $w = a_1a_2 \ldots a_n$. Let $w' = a_1a_2 \ldots a_{n-1}$. Then $decimal(w) = 2 * decimal(w') + a_n$. If $decimal(w') = 5k + r$, then $decimal(w) = 2*(5k+r)+a_n = 5*(2k)+2r+a_n$. Hence w should be in state $2r+a_n(mod\,5)$. By induction hypothesis, after reading w', the DFA will be in state r. Check that for all possible values of r (0,1,2,3,4,) and al possible a_n (0,1) , the DFA goes to the state $2r + a_n(mod\,5)$. Hence the claim.

Hence using this claim, it is obvious the the DFA accepts $A_{5,2}$, because if $w \in A_{5,2}$, then the DFA after reading w will be in state 0 which is the final state.

Handout #5

Problem 1

Construction of the new DFA M' : Let s be the start state of M. The new DFA M' has all the states of M plus the additional state s'. If s is a final state in M , then s' is a final state in M'. The start state of M' is again s. The transition function δ' of M' is :

$\forall$ states q in M, if $\delta(q, a) = q'$ and $q' \neq s$, then $\delta'(q, a) = q'$ else $\delta'(q, a) = s'$.

Finally if $\delta(s, a) \neq s$, then $\delta'(s', a) = \delta(s, a)$ else $\delta'(s', a) = s'$.

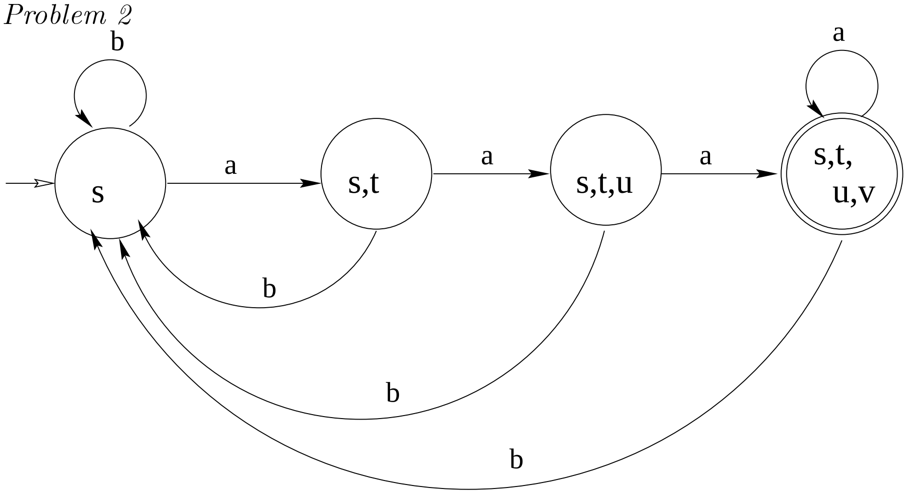
So from the construction it is obvious that in M' , the start state s has no incoming arrows.

Claim : If M reads a string w and reaches state q, then M' on w will reach q if $q \neq s$ else s'.

An immediate inference of this claim is if M' reaches a final state iff M reaches a final state on the same input. i.e M and M' are equivalent.

Proof by induction on the length of w : If $|w|$=1, then easy to see that the above claim is true. Therefore let it be true for $|w| < n$. Consider $|w| = n$. Let $w = a_1a_2 \ldots a_n$. Let $w' = a_1a_2 \ldots a_{n-1}$. $\delta'(s, w) = \delta'(\delta'(s, w'), a_n)$ Let $\delta'(s, w') = q_{n-1}$. There are 2 cases : $q_{n-1} = s'$ and $q_{n-1} \neq s'$. Consider the case $q_{n-1} \neq s'$. Then by induction hypothesis, $\delta(s, w') = q_{n-1}$. Let $\delta(q_{n-1}, a_n) = q_n$. If $q_n \neq s$, then by definition $\delta'((q_{n-1}, a_{n-1}) = q_n$ and we are done . If $q_n = s$, then by definition $\delta'((q_{n-1}, a_n) = s'$ and again we are done. The other case $q_{n-1} = s'$ is analysed similarly. Hence the claim.

Problem 2



Problem 3

The idea : A regular $\Rightarrow$ $\exists$ DFA M, such that language accepted by M is A. From M , we will construct NFA M^R such that language accepted it is A^R, the reverse of A.This implies M^R is regular.

Construction of M^R **:** Let s be the start state of M. M^R includes all the states of M plus an additional state t. The transition function δ' of M^R is deiined as follows :

$\forall$ states q in M , $\delta'(q, a)$ =includes r iff $\delta(r.a) = q$ (*reversing the edges in* M) and

$\delta'(t, \epsilon) = f \quad \forall f$ which are final states of M

t is the start state of M^R and s the final state.

Claim : Language accepted by M^R is A^R.

Proof : Suppose $w = a_1 a_2 \ldots a_n$ is acceped by M.Let M go reach the states $q_1, q_2, \cdots, q_n$ after reading $a_1, a_1a_2, \cdots, a_1a_2 \ldots a_n$ respectively. q_n is a final state. Then in M^R there is a path from t to s via $q_n, q_{n-1}, \cdots, q_1$ traversing the edges $a_n, a_{n-1}, \cdots, a_1$. Hence w^R is accepted by M^R.

Now suppose $w = a_1 a_2 \ldots a_n$ is not acceped by M. We have to show that w^R is not accpeted by M^R. On the contrary assume that w^R is accepted by M^R. This implies that starting from t we can reach s through a path consisting of edges $a_n, a_{n-1}, \cdots, a_1$. From our construction of M^R, it follows that there is a path in M from s to a final state consisting of edges $a_1, a_2, \cdots, a_n$, which implies w is accepted by M, a contradiction. Hence w^R is not accepted by M^R. Hence the claim.